# Proving Deductive Arguments Invalid by Using Counterexamples

In this lesson, you’ll be learning to

1. Use counterexamples to prove deductive arguments invalid.
2. Explain *why* counterexamples work to show that certain arguments are invalid.

To show that a deductive argument is invalid, you need show that it is logically possible that this argument could have (1) all TRUE premises but (2) still have a FALSE conclusion. So, for example, suppose that someone argues as follows: “Insofar as all serial killers are people who enjoy violent video games, we can conclude that all people who enjoy violent video games are serial killers.” We begin by putting the argument in **standard form.**

* Premise: All serial killers are people who enjoy violent video games.
* Conclusion: All people who enjoy violent video games are serial killers.

If we want to show this argument invalid, we can begin by producing a **substitution instance** that retains the logical **form**of the original argument (retains **logical** or **form** words such as “all”, “some”, “no”, “not”, “if…then”, “and”, and so on), but replaces all of the argument’s **content** (e.g., “people who enjoy violent video games”) with **variables** (such as A, B, or C).

* All S are V. // All V are S. (Logicians sometimes use “//” to mean “conclusion)

We can now produce a **counterexample** by filling in the variables in the substitution instance with new values. (So long as our new values make grammatical sense, we can use whatever values we want.)The counterexample must have TRUE premises and a FALSE conclusion. So, for example, we could produce a counterexample by letting S = dogs and V = mammals:

* All dogs are mammals (TRUE) // All mammals are dogs (FALSE)

This counterexample PROVES that *every* argument of this form (including the original argument) is invalid. Woo-hoo! So, whenever you see an argument of the form “All S are V. So, all V are S” you know that it is invalid, *regardless* of whether the premises/conclusion are true or false. The mere fact that an argument has a true conclusion and/or true premises does NOT mean it is a valid argument.

It’s important to remember that counterexamples only work as criticisms of deductive arguments, however; even the strongest inductive argument leaves open the possibility that the premises could be true and the conclusion false. (So, we *expect* inductive arguments to have counterexamples; I’ve included a few examples on the worksheet below.)

## Solved Problems

Show that the following arguments are invalid by (1) finding the logical form of each argument, and (2) producing a substitution instance of this form that serves as a counterexample.

|  |  |  |
| --- | --- | --- |
| Original Argument | Form | Counterexample |
| “If Gatsby is rich, he drives a nice car. Gatsby drives a nice car. So, Gatsby must be rich.” | If R then C. C. Therefore, R. (Fallacy of affirming the consequent.) | If pigs are birds, then pigs are animals. Pigs are indeed animals. So, pigs must be birds. |
| “If Catherine does not marry Heathcliff, she will be unhappy. Catherine will marry Heathcliff. So, she will be happy. | If not M then not H. M. Therefore, H. | If Tony the Tiger does not live in a zoo, he is not a vegetarian. Tony does live in a zoo. So, Tony the Tiger is a vegetarian. |
| “All vampires are undead creatures. All zombies are undead creatures. So, all vampires are zombies. | All V are U. All Z are U. So, all V are Z. | All horses are mammals. All dogs are mammals. So, all horses are dogs. |
| “No good tennis players skip workouts. Some people who skip workouts have high blood sugar. So, no good tennis players have high blood sugar. | No T are S. Some S are H. So, no T are H. | No dogs are cats. Some cats are animals. So, no dogs are animals. |
| “Up until today, raccoons have never been able to build and use nuclear weapons to get into garbage cans. So, raccoons won’t be able to do this tomorrow.” | “X has never occurred before. So, X will not occur today.” | “Throughout all of history, I personally have never before read a word problem about raccoons possessing nuclear weapons. So, I won’t read such a word problem today.” |

## CounterExamples! A Logic Game

Your challenge: try to come up with counterexamples for as many of the following argument forms as you can. It is easiest to generate counterexamples when you work with categories of objects that have well-defined relationships between them. For the purpose of this activity, you should be able to generate many of the counterexamples using simple groups of categories such as the following:

* Group 1: humans, females, males, mothers, fathers, sons, daughters, students, athletes, Minnesotans
* Group 2: animals, mammals, fish, dogs, cats, goldfish, walleye, wild animals, house pets

|  |  |  |
| --- | --- | --- |
|  | Argument Form | Counterexample |
|  | (Premise) If X then Y  (Premise) Y  (Conclusion) Therefore, X | If geese are reptiles, geese cannot speak fluent English. (TRUE)  Geese cannot speak fluent English. (TRUE)  Therefore, geese are reptiles. (FALSE) |
|  | All X are Y  Therefore, all Y are X |  |
|  | Some X are not Y  Therefore, some Y are not X |  |
|  | All X are Y  Therefore, all non-X are non-Y |  |
|  | Some X are Y (in logic, “Some” means “at least one”. So, “some cats are mammals” is TRUE).  Therefore, some X are not Y |  |
|  | Some X are Y  Therefore, some non-Y are non-X |  |
|  | X or Y (“or” might mean *both* are true)  X  Therefore, not Y |  |
|  | If X, then Y  Not X  Therefore, not Y |  |
|  | No M are P  No M are S  Therefore, no S are P |  |
|  | All M are P  Some S are not M  Therefore, some S are P |  |
|  | Some M are P  All S are M  Therefore, some S are P |  |
| Examples of Inductive Arguments (*every* inductive argument will have a counterexample, no matter how strong the argument is) | | |
|  | Most P are Q  The individual X is a P  So, X is a Q |  |
|  | In the past, X has always been the case.  So, in the future, X will always continue to be the case. |  |
|  | X and Y are highly correlated.  So, X causes Y |  |
|  | X has properties P and Q  Y has property P  So, Y has property Q |  |